**Mathematician or Magician?**

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*A teacher of mathematics…is perpetually faced with one overwhelming difficulty: How can he keep his students awake? …The best way…is to approach it in a spirit of play.*

 - Martin Gardner from *Mathematical Carnival*

Here are some cool math tricks you can use to engage your students in non-traditional lessons. These tricks can be great hooks into new topics, ice breakers, time wasters, or just to have in your back pocket because every minute matters in a math classroom.(NYS Principles & Standards)

Standards:

* + 5.R.4 Use standard and nonstandard representations with accuracy and detail
* **1089**
* **Boolean Magic**
	+ 4.N.13 Develop an understanding of the properties of odd/even numbers as a result of multiplication
	+ A.PS.3 Observe and explain patterns to formulate generalizations and conjectures
* **Age**
* **Missing Number**
	+ 4.N.5 Recognize equivalent representations for numbers up to four digits and generate them by decomposing and composing numbers
* **Binary Representation**
	+ 6.N.25 Evaluate expressions having exponents where the power is an exponent of one, two, or three
* **Functional Card Trick**
	+ K.A.1 Use a variety of manipulatives to create patterns using attributes of color, size, or shape
	+ 4.A.5 Analyze a pattern or a whole-number function and state the rule, given a table or an input/output box
	+ 8.R.11 Use mathematics to show and understand mathematical phenomena (e.g., use tables, graphs, and equations to show a pattern underlying a function)
	+ A2.A.37 Define a relation and function

These tricks will cover:

* Algebra
* Binary numbers
* Composition of functions
* Digital root
* Exponents, powers
* Functions
* Modulo arithmetic
* Number sense
* Parity
* Place value

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**Binary Representation**

On seven pieces of paper/cardboard copy the following sets of numbers. It is better if you mix up the digits on the card, so no one can easily see the pattern.

(set #0)

(set #1)

(set #2)

(set #3)

(set #4)

Have someone secretly choose an integer between and . Now show the person each of the five sets of numbers and ask him/her whether the number chosen is among the numbers in each given set. Remember which set(s) corresponding to a "yes" answer. (You might want to separate the pieces of paper into two piles: one for those corresponding to "yes", the other for "no"; so to easily remember which set(s) contains the secret number.) We just have them in decreasing order, so we can add in our heads.

After you have gone through all five sets of numbers, all you have to do is to add up the power of two corresponding to cards with a "yes". The sum you get is the secret number chosen.

For example, suppose the secret number is . Looking through the above table, we find that is contained in sets #1, #3, and #4. Indeed, by adding up the first number from the three sets, we get ().

The gist of this trick is based on the following theorem:

Theorem: Every positive integer has a unique binary representation, i.e., if is a positive integer, then can be uniquely written as

where is a non-negative integer and 's are either or .

Any integer can be written as a sum of different powers of .

This trick can be introduced when teaching exponents & place value.

**The Functional Card Trick**

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| **** | **** | **** | **** |
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| ****  | ****  | ****  | ****  |

Have someone pick a card from the deck, you can tell them their card!

The value of the th card from the top (face down) is:

Here, is the value of the bottom card.

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With the numerical code for suits given at the bottom of our main table, if is the suit of the bottom card, then the suit of the th card is simply:

One trick is to have a spectator cut the deck. You secretly look at the bottom card and call the card units higher in the next suit (from the "CHaSeD" sequence Clubs, Hearts, Spades, Diamonds) before revealing the top card.

This trick can be used to introduce functions in a classroom.

**1089**

Pick a -digit number where the first and last digits differ by 2 or more...

* Consider the “reverse” number, obtained by reading it backwards.
* Subtract the smaller of these two numbers from the larger one.
* Add the result to its own inverse.

This is one of the better tricks of its kind, because the effect of reversing the digits is not obvious to most people at first... If the 3-digit number reads , it's equal to and the second step gives the following result:

The quantity is between and , so the above is a 3-digit multiple of , namely: , , , , , , , or . The middle digit is always , while the first and last digits of any such multiple add up to . Thus, adding the thing and its reverse gives , as advertised.

Here are a few more tricks:

**Boolean Magic**

You have two choices. Choose *either* or ...

Multiply your chosen number by *any* odd number and multiply the number you did not choose by *any* even number. Add those two products together.

The result and the original number have the same parity (evenness or oddness).

For example, say I picked . Now multiply by an odd number, say . . Now we multiply by an even number, say . . Now add and which leads to an even number.

**Age**

1. Pick any number
2. Multiply by
3. Add
4. Multiply by
5. If you’ve had your birthday this year add , if not add
6. Subtract your birth year

Answer: The first digit of the resulting 3-digit number is your original number; the last two digits represent your age.

If we let be the number chosen in step 1, steps 2-4 give you the following:

So this first term will give us this three-digit number:

Through step 5, if you have had your birthday you add . If we add this to the from step 4:

If you have not had your birthday add giving us:

If you subtract your birth year from or , you will get your age in either case.

Thus adding the first term (), you will get the desired outcome.

**Missing Number**

1. Write a long number down using any reasonable number of digits .
2. Mix up the digits to create a second number.
3. Take these two numbers and subtract the smaller from the larger.
4. Cross off any digit you would like, this if you secret number.
5. Tell me the rest of the digits, I’ll tell you your secret number.

This is how we find the secret number. We add up the rest of your digits. If it is a two-digit number then add the digits of that number until you get a single digit number. Then you subtract the single digit number from nine. The difference is your secret number. We know this because the digital root of the difference of a number and a randomized order of the digits of that number is always nine. A number's Digital Root is found by adding all of a number's digits together. If the sum is larger than , add up the digits in the answer. Keep doing this until you have a single digit number.

We (and you!) can't tell the difference between someone crossing out a and someone crossing out a . This is why we suggest to use the digits instead of .